Subgroups- HW Problems

In problems 1-3 determine if the following sets of invertible $n \times n$ matrices with real entries are subgroups of $GL(n, \mathbb{R})$.

1. $n \times n$ matrices with determinant equal to 5.
2. Diagonal $n \times n$ matrices without zeros on the diagonal.
3. Diagonal matrices with positive numbers on the diagonal.

4. Determine if the set of real-valued, non-zero functions at every point of $\mathbb{R}$ such that $f(0) = 2$ is

a. a subgroup of all real-valued functions on $\mathbb{R}$ under addition.

b. a subgroup of all real-valued non-zero functions at every point on $\mathbb{R}$ under multiplication of functions.

5. describe all elements of the cyclic subgroup of $GL(2, \mathbb{R})$ generated by $\begin{bmatrix} 2 & 0 \\ 0 & 4 \end{bmatrix}$.

6. Which of the following groups is cyclic?

a. $G_1 = (\mathbb{Q}, +)$

b. $G_2 = (5\mathbb{Z}, +)$

c. $G_3 = \{5^n,\ n \in \mathbb{Z}\}$ under usual multiplication

d. $G_4 = (\mathbb{Z}, +)$

7. Find the order of the subgroup of $\mathbb{Z}_6$ generated by 2. What about the subgroup generated by 5?

8. Find the elements of the subgroup of $\mathbb{Z}_8$ generated by

a. 2

b. 3

c. 6

9. Suppose G is an abelian (ie commutative) group. Let A and B be subgroups of G. Prove that $AB = \{ab|\ a \in A,\ b \in B\}$ is a subgroup of G.

10. Prove that if A and B are subgroups of a group G then

$$A \cap B = \{g \in G|\ g \in A \text{ and } g \in B\}$$

is a subgroup of G.

11. Suppose G is an abelian group written multiplicatively with identity element e. Prove that $H = \{g \in G|\ g^3 = e\}$ is a subgroup of G.