Stokes' Theorem, the Divergence Theorem, and the Fundamental Theorem of Calculus- HW Problems

In problems 1 and 2 evaluate $\int_{\partial S} \omega$ directly and by Stokes' theorem.

1. $\omega = x^2 y dx + z^2 dy$

   S is the upper unit hemisphere given by

   $\vec{\Phi}(u, v) = < \cos(v)\,(\sin(u)), \sin(v)\,(\sin(u)), \cos(u) >;$

   $0 \le u \le \frac{\pi}{2}, \quad 0 \le v \le 2\pi.$

2. $\omega = (y + z)dx + (x + z)dy + (x + y)dz$

   S is the portion of the cone given by

   $\vec{\Phi}(r, \theta) = < r\cos(\theta),\ r\sin(\theta),\ r >;$

   $0 \le \theta \le 2\pi, \quad 0 \le r \le 2.$

In problems 3 and 4 evaluate $\iint_S \omega$ directly and by the divergence theorem.

3. $\omega = 3x dx dy - x dy dz$

   S is the unit sphere.

4. $\omega = z dx dy + x^2 y dy dz - y dz dx$

   S is the boundary of the solid cylinder: $x^2 + y^2 \le 4, \quad 0 \le z \le 3.$