Diagonalizability- HW Problems

For problems 1-6 determine if the matrix, A, is diagonalizable.

1. $\begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$

2. $\begin{bmatrix} 1 & 3 \\ 0 & 1 \end{bmatrix}$

3. $\begin{bmatrix} 2 & -8 \\ 1 & -4 \end{bmatrix}$

4. $\begin{bmatrix} 2 & 2 & 1 \\ 0 & 1 & 2 \\ 0 & 0 & -1 \end{bmatrix}$

5. $\begin{bmatrix} 1 & 1 & 0 \\ 0 & 1 & 1 \\ 0 & 0 & 2 \end{bmatrix}$

6. $\begin{bmatrix} 1 & 1 & 0 \\ 1 & 1 & 0 \\ 0 & 0 & 3 \end{bmatrix}$

In problems 7 and 8 determine if the linear operator T is diagonalizable. If it is, find a basis for which T is diagonal.

7. $T: P_2(\mathbb{R}) \to P_2(\mathbb{R})$ by
$T(a_0 + a_1x + a_2x^2) = 2a_0 + (-a_0 + 4a_1)x + (-3a_0 + 6a_1 + 2a_2)x^2$

8. $T: M_{2\times 2}(\mathbb{R}) \to M_{2\times 2}(\mathbb{R})$ by $T(\begin{bmatrix} a & b \\ c & d \end{bmatrix}) = \begin{bmatrix} d & b \\ c & a \end{bmatrix}$

9. Let $A = \begin{bmatrix} 5 & 6 \\ -2 & -2 \end{bmatrix}$. Find A^5 and A^{-1} by first writing $A = PDP^{-1}$ where D is a diagonal matrix.

10. Suppose $A \in M_{n\times n}(\mathbb{R})$ is diagonalizable and all of its eigenvalues are 1 or -1. Prove $A^{-1} = A$.

11. Find the values of k for which $A = \begin{bmatrix} 4 & 6 & -2 \\ -1 & -1 & 1 \\ 0 & 0 & k \end{bmatrix}$ is not diagonalizable.