

Sequences and Series of Complex Functions- HW Problems

1. Show that the sequence of functions given by $f_n(z) = \frac{1}{nz^2}$ converges uniformly on $0 < a \leq |z|$, $a > 0$, but not uniformly on $0 < |z|$.

In problems 2-4 show that the series converges uniformly in the given region.

2. $\sum_{n=1}^{\infty} \frac{z^n}{n^3}$ on $|z| \leq a < 1$

3. $\sum_{n=1}^{\infty} ne^{-nz}$ on $0 < a \leq \operatorname{Re}(z) \leq 1$

4. $\sum_{n=1}^{\infty} e^{-2n} \sin(nz)$; on $-1 < \operatorname{Im}(z) < 1$.