Bounded Linear Functionals on L^p Spaces- HW Problems

1. Show that $\|T\|_* = \sup\{T(f)\,|\,f \in X,\ \|f\| \le 1\}$.

2. Prove that $\|T\|_*$ is a norm X^*.

3. Suppose that E is measurable and $1 \le p < \infty$. Show that the set of functions in $L^p(E)$ with finite support are dense in $L^p(E)$. Show that this is not true for $L^\infty(\mathbb{R})$.