Approximations of L^p Functions- HW Problems

1. Suppose that $m(E) < \infty$ and $1 \leq p_1 < p_2 < \infty$. Is the linear space $L^{p_2}(E)$ with the norm $\| \quad \|_{p_1}$ a Banach space?

2. Suppose that E is measurable, $1 \leq p \leq \infty$, and q is the conjugate of p. Let U be a dense subset of $L^q(E)$. Show that if $g \in L^p(E)$ and $\int_E fg = 0$ for all $f \in U$ then $g = 0$.

3. Suppose that S is a dense subset of a Banach space X. Suppose that X and S have the same norm $\| \quad \|$ and that S is also Banach space with this norm. Prove that $S = X$.