Normed Linear Spaces- HW Problems

1. Let $\|f\| = \int_0^1 x^2 |f|$ for $f \in L^1[0,1]$. Show that this is a norm on $L^1[0,1]$.

2. Let $\|f\| = \int_a^b |f|$ for $f \in C[a,b]$.

a. Show that this is a norm on $C[a,b]$.

b. Show that there does not exist a $c \geq 0$ such that
$$\|f\| \leq c(\max_{a \leq x \leq b} |f(x)|) \text{ for all } f \epsilon C[a,b].$$