Functions of Bounded Variation- HW Problems

1. Prove that $f(x) = x^4 - 5x^3 + 4x^2 - 3x - 1$ is of bounded variation on $[1,2]$.

2. Prove that if f, g are of bounded variation then $f + g$ is of bounded variation.

3. For $a, b > 0$ define f on $[0,1]$ by

$$f(x) = x^a \sin\left(\frac{1}{x^b}\right) \quad \text{if } 0 < x \leq 1$$

$$= 0 \quad \text{if } x = 0.$$

a. Prove that if $a > b$ then f is of bounded variation on $[0,1]$ by showing that f' is integrable over $[0,1]$.

b. Prove if $a \leq b$ then f is not of bounded variation over $[0,1]$.

4. Suppose that φ is a step function on $[0,1]$. Find a formula for the total variation of φ.