Continuity of Integration/$L^1$ Approximations- HW Problems

1. For each function below show that $\lim_{n\to\infty} \int_1^n f$ exists, but that $f$ is not Lebesgue integrable over $[1.\infty]$.

a. $f(x) = \frac{sinx}{x}$

b. $f(x) = (-1)^n/n$ for $n \leq x < n+1$.

2. Suppose that $f$ is integrable over $E$. Show that given any $\epsilon > 0$ there is an $N \in \mathbb{Z}^+$ such that if $E_n = \{x \in E | \, |x| \geq n\}$ then $|\int_{E_n} f| < \epsilon$.