Lebesgue Outer Measure- HW Problems

1. Let $= \bigcup_{n=1}^{\infty} E_n$. Show that $m^*(E) = 0$ if and only if $m^*(E_n) = 0$ for every n.

2. If we define $rE = \{rx | \ x \in E, \ r \in \mathbb{R}\}$, what is $m^*(rE)$ in terms of $m^*(E)$? Show why.

3. Prove that the set $\{x \in \mathbb{R} | \ 0 \leq x \leq 1\}$ is uncountable using properties of outer measure.

4. Prove that if $m^*(E) = 0$ then $m^*(E \cup A) = m^*(A)$.

5. Let E be the set of irrational numbers between 0 and 1. Show that $m^*(E) = 1$.