Orthogonal Operators- HW

1. Determine which of the following matrices are orthogonal. Explain your answer.

   a. $\begin{pmatrix} 1 & 2 \\ 2 & 1 \end{pmatrix}$

   b. $\frac{1}{\sqrt{5}}\begin{pmatrix} 1 & 2 \\ -2 & 1 \end{pmatrix}$

   c. $\begin{pmatrix} \frac{1}{\sqrt{3}} & \frac{1}{\sqrt{3}} & \frac{1}{\sqrt{3}} \\ 0 & -\frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \\ \frac{2}{\sqrt{6}} & -\frac{1}{\sqrt{6}} & -\frac{1}{\sqrt{6}} \end{pmatrix}$

   d. $\begin{pmatrix} 1 & 0 & 2 \\ 0 & 3 & 0 \\ 2 & 0 & 4 \end{pmatrix}$

2. For each matrix A find an orthogonal matrix P that diagonalizes A, Ie, $P^tAP = D$. Find the matrix D as well (you can use the results of problem 2 from the previous HW assignment).

a. $A = \begin{pmatrix} 1 & 2 \\ 2 & 1 \end{pmatrix}$

b. $A = \begin{pmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{pmatrix}$

c. $A = \begin{pmatrix} 2 & 1 & 1 \\ 1 & 3 & -2 \\ 1 & -2 & 3 \end{pmatrix}$

d. $A = \begin{pmatrix} 0 & 2 & -1 \\ 2 & 3 & -2 \\ -1 & -2 & 0 \end{pmatrix}$