Normal and Self-Adjoint Operators-HW

1. Determine which of the following linear operators are normal, self-adjoint or neither. Justify you answer.
   a. $T: \mathbb{R}^2 \to \mathbb{R}^2$ by $T(a,b) = (-5b,\ 5a)$
   b. $T: \mathbb{R}^2 \to \mathbb{R}^2$ by $T(a,b) = (3a - 4b,\ 5a + 4b)$
   c. $T: \mathbb{R}^3 \to \mathbb{R}^3$ by $T(a,b,c) = ((a + 2c),\ 3b,\ (2a + 2c))$.

2. Find an orthonormal basis of eigenvectors of T for V.
   a. $V = \mathbb{R}^2$, $T(a,b) = (2a + b, a + 2b)$

   b. $V = \mathbb{R}^3$,
   $T(a,b,c) = \big((a_1 + a_2 + a_3), (a_1 + a_2 + a_3), (a_1 + a_2 + a_3)\big)$

   c. $V = \mathbb{R}^3$,
   $T(a,b,c) = ((2a_1 + a_2 + a_3), (a_1 + 3a_2 - 2a_3), (a_1 - 2a_2 + 3a_3))$.