Tangent Spaces- HW Problems

1. Let $\vec{\Phi}: \mathbb{R}^2 \to S^2 - (0,0,1) \subseteq \mathbb{R}^3$ by

$$\vec{\Phi}(u,v) = \left(\frac{2u}{u^2+v^2+1}, \frac{2v}{u^2+v^2+1}, \frac{u^2+v^2-1}{u^2+v^2+1} \right).$$

a. Find two vectors that span the tangent space of $S^2 - (0,0,1)$

at $\vec{\Phi}(1,\sqrt{2}) = \left(\frac{1}{2}, \frac{\sqrt{2}}{2}, \frac{1}{2} \right)$.

b. Write an expression for the tangent space at $\vec{\Phi}(1,\sqrt{2})$.

c. Let $\vec{v}_p = 4\vec{\Phi}_u(1,\sqrt{2}) - 8\vec{\Phi}_v(1,\sqrt{2})$ be a vector in

$T_{\left(\frac{1}{2},\frac{\sqrt{2}}{2},\frac{1}{2}\right)}(S^2 - (0,0,1))$. Let $f: S^2 - (0,0,1) \to \mathbb{R}$ by

$f(x,y,z) = xy^2 + z^3$. If $p = \left(\frac{1}{2}, \frac{\sqrt{2}}{2}, \frac{1}{2} \right)$ find $\vec{v}_p(f)$ in the

following two ways (see class notes):

i. Do the calculations in terms of u and v.

ii. Do the calculations in terms of x, y, z.

2. Let S be a surface in $\mathbb{R}^3$ given by

$\vec{\Phi}(u^1, u^2) = (u^1, u^2, (u^1)^2 + (u^2)^2)$. Let

$\vec{v}_p = \vec{\Phi}_{u^1}(u^1, u^2) + 2\vec{\Phi}_{u^2}(u^1, u^2)$ be a vector in $T_{\vec{\Phi}(u^1,u^2)}S$.

$f : S \to \mathbb{R}$, by $f(x, y, z) = x^4 + y^3 + z^2$. Find $v_p(f)$ at any point

$\vec{\Phi}(u^1, u^2)$. Your answer should be a function of only u^1, u^2 (not

x, y, z).