Contravariant and Covariant Vectors- HW Problems

1. Suppose that A is a contravariant vector on $\mathbb{R}^2$ and that

   $A = <x^1 x^2, (x^2)^2>$ in the x^1, x^2 coordinate system. Find the

   components of A in the new coordinate system given by

   $$\bar{x}^1 = (x^2)^3$$
   $$\bar{x}^2 = x^1 + 2x^2$$

   in terms of $\bar{x}^1, \bar{x}^2$.

2. Suppose that A is a contravariant vector on $\mathbb{R}^2$ and that

   $A = <x^2, x^1 x^2>$ in the x^1, x^2 coordinate system. Find the

   components of A in the $\bar{x}^1, \bar{x}^2$ coordinate system in terms of

   $\bar{x}^1, \bar{x}^2$ if

   $$x^1 = e^{\left(\bar{x}^1 + \bar{x}^2\right)}$$
   $$x^2 = e^{\left(\bar{x}^1 - \bar{x}^2\right)}.$$

3. Let $f : \mathbb{R}^2 \to \mathbb{R}$, where $f(x,y) = x^2 + y^3$. Calculate ∇f in the new

   coordinate system (r, θ) where

   $$x = r\cos\theta$$
   $$y = r\sin\theta$$

   by using the fact that ∇f is a covariant vector. Make sure your

   answer is in terms of r and θ.

4. Suppose B is a covariant vector in $\mathbb{R}^2$ and that

$B = \langle x^2, x^1 + 3x^2 \rangle$ in the x^1, x^2 coordinate system. Find B in the

new coordinate system $\bar{x}^1, \bar{x}^2$ (as in problem 1) if

$$\bar{x}^1 = (x^2)^3$$
$$\bar{x}^2 = x^1 + 2x^2$$

Make sure your answer is in terms of $\bar{x}^1, \bar{x}^2$.