Plane Curves- HW Problems

1. Find the signed curvature of each of the following plane curves:

a. $\gamma(t) = (e^t \cos(t),\ e^t \sin(t))$

b. $\gamma(t) = (a(t - \sin(t)),\ a(1 - \cos(t)));\quad a \in \mathbb{R}$.

c. $y = \cosh(x)$ (parametrize this first).

2. Consider the plane curve given by:

$$\gamma(s) = \left(\int_{t=0}^{t=s} \cos\left(\frac{t^2}{2}\right) dt,\ \int_{t=0}^{t=s} \sin\left(\frac{t^2}{2}\right) dt\right) .$$

a. Use the fundamental theorem of Calculus (i.e., $\frac{d}{ds}\int_{t=a}^{t=s} f(t)dt = f(s)$) to show that $\gamma(s)$ is a unit speed curve.

b. Find the signed curvature of $\gamma(s)$.

Note: This problem hints at a way to construct a curve γ which has curvature equal to any given smooth function.