The First Fundamental Form: Lengths of Curves on Surfaces-

HW Problems

1. Find the first fundamental form for each of the following surfaces:

a. $\vec{\Phi}(u,v) = (a(\cos(u))\sin(v), b(\sin(u))\sin(v), c(\cos(v))$ (ellipsoid)

b. $\vec{\Phi}(u,v) = (av(cos(u)),\ bv(sin(u)),\ v^2)$ (elliptic paraboloid)

c. $\vec{\Phi}(u,v) = (u(\cosh(v)),\ u(\sinh(v)),\ u^2)$ (hyperbolic paraboloid).

2a. Calculate the first fundamental form for the helicoid given by

$$\vec{\Phi}(u,v) = (vcos(u), vsin(u), u).$$

b. Use the first fundamental form to calculate the lengths of the curves on the surface where:

i) $u(t) = t,\ \ v(t) = 1;\quad 0 \le t \le 2\pi$

ii) $u(t) = t,\quad v(t) = \sin(t);\quad 0 \le t \le 2\pi$

iii) $u(t) = t,\ \ v(t) = t;\quad 0 \le t \le 2\pi$ (just write down the definite integral in this case, but don't evaluate it).

3a. Calculate the first fundamental form for the surface given by

$$\vec{\Phi}(u,v) = (u+v,\ uv,\ u).$$

b. Use the first fundamental form to write down the integral (but don't evaluate it) that represents the length of the curve on the this surface where $u(t) = t,\ \ v(t) = t^2,\quad 0 \le t \le 2.$

4a. Calculate the first fundamental form for the surface given by

$$\vec{\Phi}(u,v) = ((\cos(v))\cosh(u)\,, (\sin(v))\cosh(u)\,,\ \sinh(u))$$
$$u \in \mathbb{R} \le v \le 2\pi.$$

b. Use the first fundamental form to write down the integral (but don't evaluate it) that represents the length of the curve on the this surface where $u(t) = 2t,\ \ v(t) = t^2,\quad 2 \le t \le 4.$