# Partial Derivatives and Derivatives-HW Problems

1. Find all of the partial derivatives of the following functions.

a. $f(x, y, z) = y^z$

b. $g(x, y) = \cos[x sin y]$

c. $h(x, y, z) = z^{(x-y)}$

d. $f(x, y) = \cos(xy)$

2. Let $g: \mathbb{R}^2 \to \mathbb{R}$ by

$$g(x, y) = xy\left(\frac{x^2 - y^2}{x^2 + y^2}\right) \qquad \text{if } (x, y) \neq (0,0)$$

$$= 0 \qquad \text{if } (x, y) = (0,0).$$

a. Show that $\frac{\partial g}{\partial y}(x, 0) = x$ for all $x$ and $\frac{\partial g}{\partial x}(0, y) = -y$ for all $y$.

b. Using the limit definition of a partial derivative show that

$$\frac{\partial^2 g}{\partial x \partial y}(0,0) \neq \frac{\partial^2 g}{\partial y \partial x}(0,0).$$

3. Let $f(x, y) = x + \frac{xy}{x^2 + y^2}$ if $(x, y) \neq (0,0)$

$= 0$ if $(x, y) = (0,0)$.

i. Determine where the partial derivatives, $f_x(x, y)$ and $f_y(x, y)$, exist and find their values.

ii. Determine if $f(x, y)$ is continuous at $(0,0)$.

iii. Determine if $f(x, y)$ is differentiable at $(0,0)$.

4. Let $f(x,y) = (xy + xe^y, x\cos(y), e^{xy})$ and $h(x,y) = e^{x+y}$.
Find $Df(x,y), Dh(x,y)$, and $D(h(x,y)(f(x,y))$ at $(x,y) = (1,0)$.

5. Let $f(x,y) = \frac{xy^3}{x^3+y^6} \quad if \ (x,y) \neq (0,0)$

$$= 0 \qquad if \ (x,y) = (0,0).$$

a. Find $f_x(x,y), \ f_y(x,y)$ for all $(x,y) \in \mathbb{R}^2$.
b. Determine if $f(x,y)$ is continuous at $(0,0)$.
c. Determine if $f$ is differentiable at $(0,0)$.

6. Let $f(x,y) = (2x + 3y, xy^2)$ and
$g(u,v,w) = (e^u + v + w, e^{(v-w)} + w)$.
a. Find $Df(x,y)$ and $Dg(u,v,w)$.
b. Let $h(u,v,w) = f(g(u,v,w))$. Use the chain rule to find $Dh(0,-2,-2)$.