Limits and Continuity- HW Problems

Compute the limits if they exist. If they don't exist show why.

1. $\lim_{(x,y)\to(0,0)} \frac{xy}{x^2+y^2}$

2. $\lim_{(x,y)\to(0,0)} \frac{xy^2}{x^2+y^4}$

3. $\lim_{(x,y)\to(0,0)} \frac{3x^2y}{x^2+y^2}$

4. $\lim_{(x,y)\to(0,0)} \frac{xy}{\sqrt{x^2+y^2}}$

5. $\lim_{(x,y)\to(0,0)} \frac{x^4-4y^2}{x^4+4y^2}$

6. $\lim_{(x,y)\to(0,0)} \frac{(x+y)^2}{x^2+y^2}$

Determine the set of points where the following functions are not continuous.

7. $f(x,y) = \frac{1+x^2+y^2}{x^2-y^2}$

8. $g(x,y) = \frac{x^2-y^2}{1-x^2-y^2}$

9. $f(x,y) = \frac{x^2+y^2}{1-xy}$

10. Can the function $f(x,y) = \frac{\sin(x^2+y^2)}{x^2+y^2}$, $(x,y) \neq (0,0)$ be defined at $(x,y) = (0,0)$ in such a way that $f(x,y)$ is continuous everywhere? Explain.