

The Dot Product, Length, and Distance- HW Problems

1. Calculate $\vec{v} \cdot \vec{w}$, $\|\vec{v}\|$, and $\|\vec{w}\|$.
 - a. $\vec{v} = \langle 2, -1 \rangle$, $\vec{w} = \langle 3, 2 \rangle$
 - b. $\vec{v} = 2\vec{i} - 3\vec{j} + \vec{k}$, $\vec{w} = -4\vec{i} + \vec{j} + 3\vec{k}$

2. Find a unit vector in the direction of $\vec{v} = 3\vec{i} - 2\vec{j} + \vec{k}$.

3. Find all values of x such that the vectors $\langle x, 2, -5 \rangle$ and $\langle x, 2, x \rangle$ are perpendicular (ie orthogonal).

4. Let $\vec{v} = \langle 1, -2, 2 \rangle$ and $\vec{w} = \langle 3, 1, 2 \rangle$. Find the vector projection of $\vec{v}$ on to $\vec{w}$ and the vector projection of $\vec{w}$ on to $\vec{v}$.

5. Show that $\vec{v} = 4\vec{i} - 2\vec{j} + 8\vec{k}$ and $\vec{w} = -6\vec{i} + 3\vec{j} - 12\vec{k}$ are parallel using the dot product.

6. Let $\vec{v} = \langle 1, -2, 2 \rangle$ and $\vec{w} = \langle 3, 1, 2 \rangle$. Show that $\vec{v}$ and $\vec{w}$ satisfy:
 - a. The Cauchy-Schwarz inequality: $|\vec{v} \cdot \vec{w}| \leq \|\vec{v}\| \|\vec{w}\|$.
 - b. The triangle inequality: $\|\vec{v} + \vec{w}\| \leq \|\vec{v}\| + \|\vec{w}\|$.