

Limits of Functions- HW Problems

1. Prove the following limits from the δ, ϵ definition of

$$\lim_{x \rightarrow p} f(x) = q.$$

a. $\lim_{x \rightarrow 2} (3x - 4) = 2$

b. $\lim_{x \rightarrow 4} (x^2 - 2x) = 8$

c. $\lim_{x \rightarrow 5} \left(\frac{x^2 - 25}{x - 5} \right) = 10$

d. $\lim_{x \rightarrow 2} (x^3) = 8$

e. $\lim_{x \rightarrow \frac{1}{2}} \frac{1}{x} = 2$

f. $\lim_{x \rightarrow 0} (\sqrt[3]{x}) = 0$

g. $\lim_{(x,y) \rightarrow (0,0)} (x^2 - 2y^2 + 1) = 1$

2. Prove (hint: use the squeeze theorem):

a. $\lim_{x \rightarrow 0} \left(x \cos\left(\frac{1}{x}\right) \right) = 0$

b. $\lim_{x \rightarrow 0} (x \tan^{-1}(x)) = 0$

3. Assume $\lim_{x \rightarrow a} f(x) = L$. If $f(x) \geq k$, prove that $L \geq k$. (Hint: assume that $L < k$ and use a δ, ϵ argument to get a contradiction).

4. Using a δ, ϵ argument prove that $\lim_{x \rightarrow 4} (\sqrt{x}) = 2$.

Hint: $\sqrt{x} - \sqrt{a} = [\sqrt{x} - \sqrt{a}][(\sqrt{x} + \sqrt{a})/(\sqrt{x} + \sqrt{a})]$
 $= (x - a)/(\sqrt{x} + \sqrt{a}), \quad \text{where } a = 4.$