Differentiation- HW Problems

1. Determine where $f'(x)$ exists, an expression for it, and where $f'(x)$ is continuous:

a. $f(x) = x^3 \sin\left(\frac{1}{x^2}\right) \qquad x \neq 0$

$= 0 \qquad\qquad x = 0$

b. $f(x) = x^4 \sin\left(\frac{1}{x^2}\right) \qquad x \neq 0$

$= 0 \qquad\qquad x = 0$

c. $f(x) = \frac{e^{-(\frac{1}{x^2})}}{x} \qquad x \neq 0$ (the exponent of e is $-\frac{1}{x^2}$)

$= 0 \qquad\qquad x = 0$

d. $f(x) = |x|^3 \quad x \epsilon \mathbb{R}$ (recall: $|x|^3 = x^3$ if $x \geq 0$

$= -x^3$ if $x < 0$).

2. Let $f(x) = x^a$ for $x > 0$

$= 0$ for $x \leq 0$.

a. For what values of "a" is $f(x)$ continuous at $x = 0$? Justify your answer.

b. For what values of "a" is $f(x)$ differentiable at $x = 0$? Justify your answer.