Continuity and Connectedness- HW Problems

1. Prove $f(x)$ has a zero (i.e., a point where $f(p) = 0$) on each interval. You can just assert that the functions are continuous on the relevant intervals.

a. $f(x) = x^2 + 3x - 2;$ on $[0,2]$

b. $f(x) = -2e^{-x}(\cos(2x))$ on $[0,\frac{\pi}{2}]$

c. $f(x) = (x^2 + 3x - 2)\ln(x^2 + 4)$ on $[0,1]$

2. Let $f(x) = xe^x$. Prove there is a point $p\epsilon[0,2]$ where $f(p) = e$.

3. Let $f(x), g(x)$ be continuous on the interval $[a, b]$. Suppose $f(a) < g(a)$ and $f(b) > g(b)$. Prove there is a point p in the interval $[a, b]$ where $f(p) = g(p)$.

4. A function f is said to have a "fixed point" if there is some point p where $f(p) = p$. Show that the function $f(x) = \cos x$ has a fixed point in the interval $[0,\frac{\pi}{2}]$.

5. A function f is said to have a "fixed point" if there is some point p where $f(p) = p$. Suppose $f:[0,1] \rightarrow [0,1]$ and is continuous at every point of $[0,1]$. Prove that f has a fixed point.