Cyclic Groups- HW Problems

1. Find the greatest common divisor (GCD) of 108 and 360.

2. Find the number of generators of a cyclic group of order 16.

In problems 3-5 find the number of elements in the cyclic group.

3. The cyclic subgroup of $\mathbb{Z}_{24}$ generated by 15.

4. The cyclic subgroup of $\mathbb{Z}_{54}$ generated by 21.

5. The cyclic subgroup of $\mathbb{Z}_{35}$ generated by 14.

Find the subgroups of the given group and all generators of those subgroups. Draw a subgroup diagram.

6. $\mathbb{Z}_{12}$

7. $\mathbb{Z}_{18}$

Find all orders of subgroups of

8. $\mathbb{Z}_{16}$

9. $\mathbb{Z}_{19}$

10. Let $a, b \in \mathbb{Z}^+$. Show that $H = \{na + mb \mid n, m \in \mathbb{Z}\}$ is a subgroup of $\mathbb{Z}$.

11.   Suppose p and q are distinct prime numbers. Find the number of generators of $\mathbb{Z}_{pq}$.

12.   Indicate whether the following statements are true or false.

a.   Every subgroup of a cyclic group is cyclic.

b.   Every group with at least two elements has at least 2 subgroups.

c.   Every abelian group is cyclic.

d.   Every cyclic group is abelian.

e.   Every infinite cyclic group must have an infinite number of generators.