Field Extensions- HW Problems

In problems 1-3 show that $a \in \mathbb{C}$ is algebraic over $\mathbb{Q}$.

1. $a = 1 - \sqrt{2}$
2. $a = 1 - i$
3. $a = \sqrt{1 + \sqrt[3]{3}}$

In problems 4 and 5 find an irreducible polynomial over $\mathbb{Q}$ where a is a root of that polynomial.

4. $a = \sqrt{3 - \sqrt{6}}$
5. $a = \sqrt{2} - i.$

6a. Show that $x^2 - 2$ is irreducible in $\mathbb{Z}_3[x]$.

b. Let a be a zero of $x^2 - 2$ in an extension field of $\mathbb{Z}_3$. Find the multiplication table for the (9) elements of $\mathbb{Z}_3(a)$, $0, 1, 2, a, 2a,$ $1 + a, 1 + 2a, 2 + a, 2 + 2a.$

In problems 7-13 determine if the statements are true or false.

7. π is transcendental over $\mathbb{R}$.
8. e is transcendental over $\mathbb{Q}$.
9. $\mathbb{C}$ is a simple field extension of $\mathbb{R}$.

10. Every field F contains an element $a \in F$ such that a is not algebraic over F.

11. $\mathbb{R}$ is an extension field of $\mathbb{Z}_5$.

12. Every nonconstant polynomial over a field F has a root in some field extension of F.

13. $\sqrt{\pi}$ is algebraic over $\mathbb{R}$.