Factoring Polynomials over a Field- HW Problems

1. Find $q(x)$ and $r(x)$ if $f(x) = g(x)q(x) + r(x)$ and $f(x) = x^5 + 3x^4 + 5x^2 - 3x + 2$ and $g(x) = 3x^2 + 4x - 2$ in $\mathbb{Z}_7[x]$.

2. Find all generators of the cyclic groups $\mathbb{Z}_7^*$, $\mathbb{Z}_{11}^*$, $\mathbb{Z}_{13}^*$ under multiplication.

In problems 3-5 factor the polynomial into linear factors over the given ring.

3. $x^3 + 4x^2 + x + 4$ in $\mathbb{Z}_5[x]$.

4. $x^3 + 5x^2 + 2x + 6$ in $\mathbb{Z}_7[x]$.

5. $x^3 - 2x^2 - x + 2$ in $\mathbb{Z}_5[x]$.

In problems 6 and 7 factor the polynomials into irreducible factors.

6. $x^3 + 4x + 4$ in $\mathbb{Z}_5[x]$.

7. $x^3 + 5x^2 + x + 6$ in $\mathbb{Z}_7[x]$.

8. Show that $x^2 + 6x + 2$ is irreducible over $\mathbb{Q}$. Is it irreducible over $\mathbb{R}$? Explain.

9. Show that $x^3 + 2x^2 + 3x - 4$ is irreducible over $\mathbb{Q}$.

10. Use Eisenstein's criterion to show that $x^7 + 48x - 24$ is irreducible over $\mathbb{Q}$.

11. Find all irreducible polynomials of degree 3 in $\mathbb{Z}_2[x]$.